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DESIGN CHARTS FOR AXIAL-FLOW COMPRESSORS
HAVING CONSTANT ROTOR WORK DISTRIBUTION
OVER THE BLADE SPAN

ROBERT E. HUNTER

AERONAUTICAL RESEARCH LABORATORY

APRIL 1953

WRIGHT AIR DEVELOPMENT CENTER

NOTICES

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DESIGN CHARTS FOR AXIAL-FLOW COMPRESSORS HAVING CONSTANT ROTOR WORK DISTRIBUTION OVER THE BLADE SPAN

Robert E. Hunter

Aeronautical Research Laboratory

April 1953

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Wright Air Development Center
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United States Air Force
Wright-Patterson Air Force Base, Ohio

FOREWORD

This report was prepared by the Aerodynamics Research Branch of Aeronautical Research Laboratory, Directorate of Research, to supplement facilities research projects as authorized by Research and Development Order No. 465-6, "Facility Research." Dr. H. Von Ohain served as project engineer.

Grateful acknowledgement is made to Mr. M. O. Lawson for his assistance in the preparation of this report.

ABSTRACT

This technical report attempts to provide a means of shortening the compressor design procedure through the use of charts. To facilitate a more rapid procedure, the charts are restricted in application to constant rotor work axial-flow compressors.

The design charts give the relationships between the mean radius velocity triangle and compressor performance and dimensions. Ranges are established for the variables to include all normal design problems and also to rule out almost immediately all impractical solutions.

The charts are generalized with respect to atmospheric conditions to permit greater applicability. That is, the charts may be used for design problems that involve initial atmospheric conditions other than standard and also for multi-stage compressors by application to each stage in succession.

Relationships are established between the mean radius velocity triangle and certain specific compressor parameters such as pressure coefficient, flow coefficient, throttle number and specific speed.

Use of the charts is illustrated by the solution of a hypothetical design problem. References are given which provide various methods for determining the blade profiles.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

LESLIE B. WILLIAMS
Colonel, USAF
Chief, Aeronautical Research
Laboratory
Directorate of Research

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Nomenclature

ΔC_v — Tangential component of the airflow velocity imparted by the rotor at the mean radius.

U — Tangential velocity of the rotor blade section at the mean radius.

V — Axial velocity component of the airflow.

W_m — Mean velocity relative to the rotor blade at the mean radius.

a — $\Delta C_v / W_m$, blade load factor.

b — U / W_m , dimensionless rotational speed.

d — $V / W_m = \sin \beta$, dimensionless axial velocity.

β — Flow angle.

G — Weight flow per second.

R — Rotor tip radius.

γ — Hub ratio.

ρ — Air density.

g — Acceleration of gravity.

H — Pressure head or work per unit weight.

P — Power expressed in horsepower units.

ω — Rotor angular velocity in radians per second.

N — Rotor rotational speed in RPM.

Δp — Total pressure increase.

η — Efficiency.

C_s — Speed of sound.

θ — The ratio of absolute ambient temperature to absolute standard temperature.

δ — The ratio of ambient pressure to standard pressure.

ϕ — Flow coefficient.

ψ — Pressure coefficient.

N_T — Throttle number.

N_s — Specific speed.

r — Any radius.

A_A — Annulus area.

h — Blade length.

υ — Volume flow.

C_L — Lift coefficient.

σ — Solidity.

Subscripts

s — Values converted to standard atmospheric conditions.

n — Values converted to both standard atmospheric conditions and 1-hp power input.

R — Values taken at the tip section.

H — Values taken at the hub section.

r — Values taken at any radius.

DESIGN CHARTS FOR AXIAL-FLOW COMPRESSORS

HAVING CONSTANT ROTOR WORK DISTRIBUTION

OVER THE BLADE SPAN

Introduction

The process of designing turbo-compressors is usually long and involved. This report attempts to facilitate a rapid design procedure by providing charts which yield final design values and eliminate impractical solutions almost immediately.

One of the most common turbo-compressor types is the axial-flow compressor with which this report is concerned. All single-stage and most multi-stage axial-flow compressors are of the constant rotor work type and, hence, consideration of this type alone will maintain a reasonable degree of generality.

SECTION I

Procedure

Since this study is restricted to constant rotor work axial-flow compressors, the velocity triangle for any point along the rotor blade span is determined by any one velocity triangle if the type of pre-rotation is known. In this study the mean radius velocity triangle will be taken as being representative of the entire blade. Limitations will be applied to this mean radius velocity triangle in order to establish practical ranges for the charts. Insofar as some of these limitations would normally be applied at the blade root and others at the tip, the chart ranges will be made flexible so as to avoid over-restricting the charts.

Relationships will be established between the mean radius velocity triangle and compressor performance and dimensions. These relationships constitute the basic design chart equations. Furthermore, relationships between the mean radius velocity triangle and certain specific compressor parameters such as flow coefficient, pressure coefficient, throttle number and specific speed will be given.

SECTION II

Chart Equations and Compressor Parameters

The equations used in plotting the charts give relationships between the mean radius velocity triangle and the compressor performance and geometry.

The compressor performance is given by the power-input, P, the rotational speed, N, and the total pressure increase, Δp. The compressor geometry is given by the rotor radius, R, and the hub ratio, ν. Three velocity triangle parameters which are sufficient for determining the geometry of the mean radius velocity triangle and which will be used for the forthcoming equations are as follows:

The blade load factor is:

$$a = \frac{\Delta C_v}{W_m}$$

The dimensionless rotational speed is:

$$b = \frac{U}{W_m}$$

The dimensionless axial velocity is:
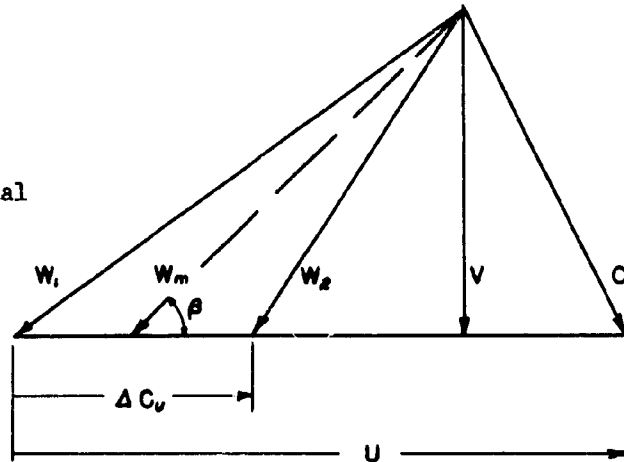
$$d = \frac{V}{W_m} = \text{SIN } \beta$$

FIGURE I

The equations relating these eight compressor parameters are as follows:

$$R = \left[\frac{2217\ P\,b^2}{\rho N(1-\nu)(1+\nu)^2 a\,d} \right]^{1/3} \quad\quad\quad (I)$$

$$\frac{\Delta p}{\rho} = 0.0598 \left[\frac{P^2 \rho^2 N^4 (1+\nu)^2 a^3}{(1-\nu)^2 d^2 b} \right]^{1/3} \quad\quad\quad (III)$$

$$\frac{\Delta p}{q} = 0.456 \frac{P^2 \rho_a\, b}{R(1-\gamma^2)^2 d^2} \tag{II}$$

These are the basic design chart equations relating the ten compressor variables. Normally, the density, power, rotational speed and total pressure increase (or weight flow) are given in a design problem. The chart equations are capable of determining two unknowns (the derivation given in Appendix II shows that the three chart equations are derived from two independent equations), thus leaving four unknowns to be selected by the designer. The charts provide a rapid graphical solution of the above three equations and aid considerably in the selection of the four remaining unknown quantities. A specific example of the use of the charts is given in Appendix I and the derivation of the three chart equations is given in Appendix II.

Two specific parameters which are of importance to both single and multi-stage compressors are the pressure coefficient, ψ, and the flow coefficient, ϕ. These two coefficients are characteristic of compressors independent of the rotational speed if the Mach number is sufficiently low and the Reynolds number is sufficiently high.

The pressure coefficient is expressed in terms of the efficiency, the hub ratio, the blade load factor, and the dimensionless rotational speed as follows:

$$\psi = \eta \frac{(1+\gamma)^2 a}{2b} \tag{IV}$$

The flow coefficient is expressed in terms of the hub ratio, the dimensionless rotational speed and the dimensionless axial velocity as follows:

$$\phi = \frac{(1+\gamma)d}{2b} \tag{V}$$

As an aid in selecting certain geometric properties such as number of blades and hub ratio in such a way that the total-energy losses become a minimum, a dimensionless characteristic parameter known as specific speed is useful. It is well known through its application to water turbines. Specific speed characterizes the type machine by its performance values (speed, volume/sec., total head) under optimum efficiency conditions. The optimum efficiency conditions are, however, affected by excessive Mach numbers and low Reynolds number values.

Specific speed is seldom considered in the design of multi-stage axial-flow compressors but is particularly applicable to single-stage axial-flow compressors.

The specific speed can be expressed in terms of the mean radius velocity triangle and the hub ratio as follows:

$$n_s = \left[\frac{(1-\nu)d(2b)^{\frac{1}{2}}}{(1+\nu)a^{\frac{3}{2}}}\right]^{\frac{1}{2}} \qquad \text{(VI)}$$

Another characteristic number, introduced by Keller, is the throttle number. Keller considered a single-stage axial-flow compressor consisting of a rotor and a stator. He defined throttle number as the ratio of the kinetic energy content of the flow behind the stage to the total rotor work.

Keller found that the throttle number has an important influence upon the optimum efficiency of a single-stage axial-flow compressor in that it indicates the amount of kinetic energy behind the stage which must be transformed into static pressure by a diffuser. For instance if the throttle number is less than one, the static pressure behind the stage is greater than atmospheric. If the throttle number is one, the compressor will act as a fan without a diffuser. If the throttle number is greater than one, a diffuser must be employed to raise the static pressure above atmospheric level before flow will occur.

The throttle number can be expressed in terms of the mean radius velocity triangle and hub ratio as follows:

$$N_T = \frac{d^2}{2\eta ab} \qquad \text{(VII)}$$

The derivations of the expressions for the above specific compressor parameters are given in Appendix III.

SECTION III

Chart Limitations

There exist certain practical limitations for the individual quantities contained in the design charts which should be considered so as to establish the ranges of the charts. It is to be noted that these limitations are by no means absolute, unvarying quantities. The limitations

are influenced by so many parameters that methods do not exist for establishing them definitely. For this reason, the limitations proposed represent generally accepted values.

The quantity a is given the range 0 to 0.6 on the basis of Keller's investigation of cascade flow (Axial Flow Fans, 1937). Keller found that to avoid stalling, the product of solidity and lift coefficient should not exceed 1.1. This product, $C_L\sigma = 2\Delta C_u/W_m$, corresponds to $2a$. The above restriction places a maximum value of 0.6 on a.

Keller, however, conducted his investigation at one value of Reynolds number. It has since been shown that permissible solidity values increase with increasing Reynolds number. Because of this consideration, the range of a is extended to 0.8.

The quantity d is given the range 0.5 to 0.9. This corresponds to a range in flow angle of 30° to 65° which will include all normal conditions.

The quantity b is assigned the range 0.25 to 1.5 to include all practical values of positive and negative prerotation.

The hub ratio, ν is given the range 0.4 to 0.9. Hub ratios of less than 0.4 are impractical for constant rotor work type compressors. Hub ratios of more than 0.9 are impractical because gap losses become excessive.

The rotational speed, N, is given the range 100 rpm to 100,000 rpm which will include all normal operational values for most compressors.

The ranges of Δp and tip radius, R, are determined from the above ranges and are found to be as follows:

$$\Delta p = 0.001 \text{ psi to 5 psi}$$

$$R = 0.5 \text{ to 30 inches}$$

Charts I, II, and III are plotted from equations I, II, and III taking into account the limitations given above. Dashed curves are plotted in excess of the ranges given above to include values which are possible but improbable under normal circumstances.

Mach number limitations are not considered in determining the chart ranges. These should be considered after establishing the velocity triangles.

SECTION IV

Generalizing the Charts

The design charts are plotted for standard atmospheric conditions and 1 hp power-input. Nearly all design problems have other atmospheric and power-input conditions and, therefore, expressions must be found for converting the design values to normal values so that the design charts may be of general use.

The quantities a , b , d , and r are dimensionless geometric properties of the compressor and, therefore are not affected by changes in atmospheric and power-input conditions. The remaining quantities (P , R , N , and Δp) must be corrected.

A detailed discussion of the generalization procedure is given in Appendix IV.

To facilitate easier use of the design charts, the scales are labeled as functions of the actual conditions rather than standard conditions.

Generalizing the charts with respect to atmospheric conditions permits using the charts in the design of multi-stage compressors by considering the exit conditions of one stage as being the inlet conditions of the following stage.

APPENDIX I

Use of the Charts

Assume a single-stage axial-flow compressor given with a power source which developes 10 hp at 3500 revolutions per minute and a hub ratio of 0.75. It is desired to obtain an increase in total pressure of 0.25 psi at an atmospheric temperature of 40°F and an atmospheric pressure of 13 psi. Furthermore, it is desired to obtain the afore-mentioned values without utilizing a stator in front of the rotor or a diffuser (for the purpose of establishing flow) at the rear.

The values of δ and θ are first determined from the given atmospheric conditions.

$$\delta = \frac{13.0}{14.7} = 0.884$$

$$\theta = \frac{499.4}{518.4} = 0.963$$

Since the values of pressure increase and rotational speed are given, Chart III will be used first to determine the mean radius velocity triangle.

Start in the fourth quadrant by computing the following values:

(a) $\dfrac{NP^{\frac{1}{2}}}{\theta^{\frac{3}{4}}\delta^{\frac{1}{2}}} = \dfrac{3500\,(10)^{\frac{1}{2}}}{(0.963)^{\frac{3}{4}}(0.884)^{\frac{1}{2}}} = 12,110$

Assume an efficiency, $\eta = 0.85$

(b) $\dfrac{\Delta p}{\eta\,\delta} = \dfrac{0.25}{0.85 \times 0.884} = 0.333$

Values (a) and (b) together locate point 1 in the fourth quadrant of Chart III. A horizontal line from point 1 to the third quadrant curve $\gamma = 0.75$ locates point 2.

A vertical line from point 2 into the second quadrant cuts the curves corresponding to values of a ranging from 0.16 to 0.45. Drawing horizontal lines from the zero prerotational lines (established by the requirement that there is to be no stator in front of the rotor) into the second quadrant further limits the possible values of a. Successive application of this procedure yields a possible range of a from 0.26

to 0.35. Selecting the value $a = 0.3$ locates point 3. A horizontal line from point 3 to the zero prerotation curve in the first quadrant locates point 4. The values of b and d are given by the Chart at point 4.

$$b = 0.95$$

$$d = 0.6$$

The next step is to determine if the additional condition of no diffuser is satisfied. This is done by considering the throttle number which is evaluated from equation (VII) as follows:

$$N_T = \frac{d^2}{2\eta ab} = \frac{0.36}{2 \times 0.85 \times 0.3 \times 0.95} = 0.743$$

The value of the throttle number is seen to be sufficiently less than unity to insure that no diffuser will be necessary to establish flow. Hence, the values of a, b, and d determined above are acceptable. (A diffuser would be needed, however, if it were desired to transfer more of the energy into static pressure.)

The value of the rotor radius is now found from Chart II using the previously determined values of a, b, and d. The procedure is shown on Chart II. The value of $(R\theta^{\frac{1}{2}}\delta^{\frac{1}{2}}P^{\frac{1}{2}})$ is seen to be 2.76, from which R is found as follows:

$$R = 2.76\,\theta^{-\frac{1}{2}}\delta^{\frac{1}{2}}P^{\frac{1}{2}} = 2.76(0.963)^{-\frac{1}{2}}(0.884)^{\frac{1}{2}}(0.5)^{\frac{1}{2}}$$

$$R = 9.37 \text{ INCHES}$$

The mean radius is:

$$R_m = R \frac{(1+\nu)}{2} = 9.37 \times \frac{1.75}{2} = 8.20 \text{ INCHES}$$

The annulus area is:

$$A_A = \pi R^2 (1-\nu^2) = \pi \frac{(9.37)^2}{144}(1 - 0.75^2) = 0.838 \text{ FT}^2$$

The blade length is:

$$h = R(1-\nu) = 9.37 \times 0.25 = 2.34 \text{ INCHES}$$

The quantities needed to construct the mean radius velocity triangle are found as follows:

$$U = \frac{2\pi R_m N}{12 \times 60} = 250 \text{ F.P.S.}$$

$$W_m = U/b = 250/0.95 = 263 \text{ F.P.S.}$$

$$\Delta C_u = W_m \Omega = 263 \times 0.3 = 78.9 \text{ F.P.S.}$$

$$V = W_m d = 263 \times 0.6 = 157.8 \text{ F.P.S.}$$

The quantities necessary for determining the velocity triangles at the rotor hub and tip (denoted by the subscripts H and R respectively) are found as follows:

$$U_R = UR/R_m = 250 \times 9.37/8.20 = 286 \text{ F.P.S.}$$

$$U_H = \gamma U_R = 0.75 \times 286 = 214 \text{ F.P.S.}$$

Since the compressor is of the constant rotor work type, $r \Delta C_{u_r} = $ constant, and:

$$\Delta C_{u_R} = \Delta C_u R_m/R = 78.9 \times 8.20/9.37 = 69.0 \text{ F.P.S.}$$

$$\Delta C_{u_H} = \Delta C_{u_R}/\gamma = 69.0/0.75 = 92.0 \text{ F.P.S.}$$

$$W_{m_R} = \sqrt{V^2 + (U_R - \tfrac{1}{2}\Delta C_{u_R})^2} = \sqrt{157.8^2 + (286 - 34)^2}$$

$$W_{m_R} = 297 \text{ F.P.S.}$$

$$W_{m_H} = \sqrt{V^2 + (U - \tfrac{1}{2}\Delta C_{u_H})^2} = \sqrt{157.8^2 + (214 - 46)^2}$$

$$W_{m_H} = 231 \text{ F.P.S.}$$

The velocity triangles for the hub, mean radius, and tip section are shown in Figure 2.

At this point it is appropriate to compute the tip Mach number. The relative intake velocity at the tip is found as follows:

$$W_{1_R} = \sqrt{V^2 + U_R^2} = \sqrt{157.8^2 + 286^2}$$

$$W_{1_R} = 327 \text{ F.P.S.}$$

The speed of sound is found from the expression which gives the
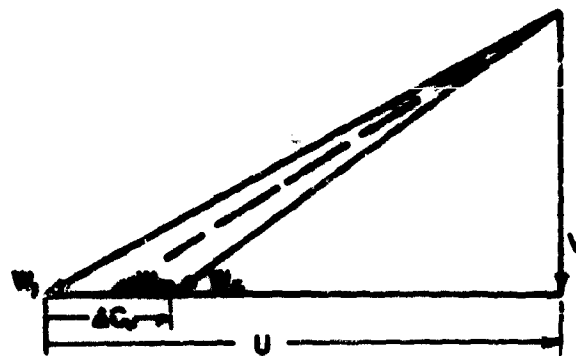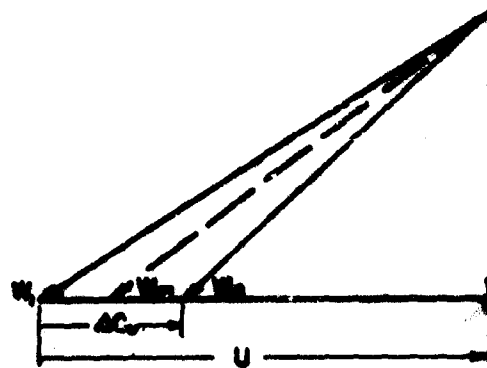
FIG. 2(a) TIP VELOCITY TRIANGLE
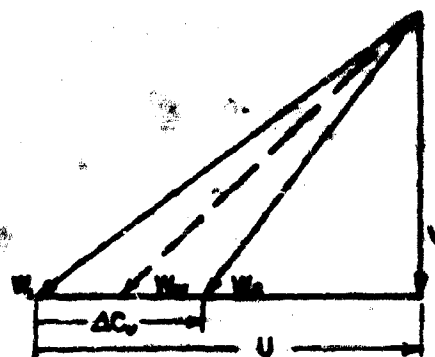

FIG. 2(b) MEAN VELOCITY TRIANGLE

FIG. 2 (c) HUB VELOCITY TRIANGLE

FIG. 2 VELOCITY TRIANGLES FOR THE EXAMPLE PROBLEM

relationship between sonic velocity and atmospheric temperature.

$$C_s = 1116 \sqrt{\theta} = 1116\sqrt{0.963} = 1075 \text{ F.P.S.}$$

and the Mach number is:

$$M_R = 327/1075 = 0.30$$

As is usually the case, the Mach number is found to be far less than critical. Hence, Mach number effects are negligible.

The volume flow is found as follows:

$$U = A_A V = 0.838 \times 157.8 = 132.2 \text{ FT}^3/\text{SEC}$$

To avoid adverse effects due to low Reynolds number, a Reynolds number of 300,000 is assumed at the mean radius. This results in a blade chord length of 2.4 inches. Assuming a lift coefficient of 0.5 at the mean radius, the solidity is determined by the load factor equation to be 1.2. The number of blades is now found to be 26.

If so desired, the values of pressure coefficient, flow coefficient and specific speed may be determined from equations (IV),(V) and (VI) respectively.

Determining the blade profiles is not within the scope of this report. However, references for methods of doing this are given below.

      (a) The Betz Method

      (b) F. Weinig, "Flow Around Turbine and Compressor Blades", BuShips, Navy Dept. 1946.

      (c) Information of the Aerodynamic Design of Axial-Flow Compressors in Germany, Navy Dept. October 1945.

      (d) H. Constants Rule, "The Present Basis for Axial-Flow Compressor Design" Part I, Aero Research Council, R. & M., June, 1942, A. R. Howell.

Had the compressor under consideration been multi-stage rather than single-stage the above procedure would be repeated for each stage in succession.

APPENDIX II

Derivation of the Chart Equations

To facilitate a more easily followed derivation, the mean radius velocity triangle parameters are repeated.

The blade load factor, $a = \Delta C_u / W_m$

The dimensionless rotational speed, $b = U / W_m$

The dimensionless axial velocity, $d = V / W_m$

The weight flow through the compressor is given by the following equation:

$$G = \pi R^2 (1-\gamma^2) V \rho g$$

and in terms of the dimensionless axial velocity is:

$$G = \pi R^2 (1-\gamma^2) d W_m \rho g$$

The pressure head or work per unit weight is given by the equation

$$H = U \Delta C_u / g$$

and in terms of the blade load factor and dimensionless rotation speed it is

$$H = \tfrac{1}{g} a b (W_m)^2$$

The power input to the compressor is given by the equation

$$P = GH = \pi R^2 (1-\gamma^2) d (W_m)^3 \rho a b \qquad (1)$$

W_m , in the above equation, can be expressed in terms of the compressor geometry and the mean radius velocity triangle parameters in the following manner.

The angular velocity, ω is
$$\omega = \frac{U}{R_m} = \frac{2U}{R(1+\gamma)}$$
and in terms of the dimensionless rotational speed, $\omega = \frac{2 W_m b}{R(1+\gamma)}$

and in turn, the expression for the rotational speed, N, is

$$N = \frac{19.1\,W_m\,b}{R(1+\nu)}$$

and

$$W_m = \frac{NR(1+\nu)}{19.1\,b} \qquad (2)$$

Introducing the expression for W_m into equation (1)

$$P = \pi R^2(1-\nu^2)d\,\frac{N^2R^2(1+\nu)^3}{(19.1)^2 b^2}\,\rho a b$$

or,

$$P = 4.51 \times 10^{-4}\,\rho R^4 N^2(1-\nu)(1+\nu)^4 \frac{ad}{b} \qquad (3)$$

and solving for R

$$R = \left[\frac{2217\,Pb^2}{\rho N^2(1-\nu)(1+\nu)^4 a d}\right]^{1/4} \qquad (I)$$

The expression for the pressure increase, Δp, is as follows:

$$\Delta p = \eta\,\rho\,U\,\Delta C_U$$

and in terms of the blade load factor, dimensionless rotational speed and equation (2)

$$\frac{\Delta p}{\eta} = \rho a\,\frac{N^2R^2(1+\nu)^2}{(19.1)^2\,b}$$

or

$$\frac{\Delta p}{\eta} = \frac{\left[4.51 \times 10^{-4}\,\rho R^4 N^2(1-\nu)(1+\nu)^4 \frac{ad}{b}\right]b}{(19.1)^2 \times 4.51 \times 10^{-4}\,R^2N(1-\nu)(1+\nu)^2 d} \qquad (4)$$

The terms inclosed in brackets can be seen from equation (3) to be P. Hence,

$$\frac{\Delta p}{\eta} = \frac{\left[P^{3/4}(4.5 \times 10^{-4})^{1/4}\rho^{1/4}R N^{1/2}(1-\nu)^{1/4}(1+\nu)(\frac{ad}{b})^{1/4}\right]b}{(19.1)^2 \times 4.51 \times 10^{-4}\,R^2N(1-\nu)(1+\nu)^2 d}$$

canceling N and collecting terms

$$\frac{\Delta p}{\eta} = \frac{0.0767\, P^4 \rho^{\frac{1}{3}} a^{\frac{4}{3}} b^{\frac{4}{3}}}{0.1645\, R^4 (1-\nu)^{\frac{1}{3}} (1+\nu)^{\frac{4}{3}} d^{\frac{4}{3}}}$$

or,

$$\frac{\Delta p}{\eta} = 0.466 \left[\frac{P^4 \rho\, a\, b}{R^4 (1-\nu)^2 d^2} \right]^{\frac{1}{3}} \tag{II}$$

The following equation is derived from equation (4)

$$\frac{\Delta p}{\eta} = \frac{\left[P^4 (4.51 \times 10^{-6})^{\frac{3}{2}} R^2 N^{\frac{3}{2}} (1-\nu)^{\frac{3}{2}} (1+\nu)^{\frac{1}{2}} \left(\frac{ad}{b}\right)^{\frac{3}{2}} \right] b}{(19.1)^2 \times 4.51 \times 10^{-6} R^2 N (1-\nu)(1+\nu)^2 d}$$

upon elimination of the radius the equation becomes

$$\frac{\Delta p}{\eta} = \frac{9.83 \times 10^{-3} P^{\frac{4}{3}} \rho^{\frac{2}{3}} N^{\frac{3}{2}} (1+\nu)^{\frac{3}{2}} a^{\frac{2}{3}}}{0.1645 (1-\nu)^{\frac{2}{3}} d^{\frac{4}{3}} b^{\frac{1}{3}}}$$

or,

$$\frac{\Delta p}{\eta} = 0.0598 \left[\frac{P^2 \rho^3 N^4 (1+\nu)^3 a^3}{(1-\nu) d^2 b} \right]^{\frac{1}{3}} \tag{III}$$

Equations (I), (II), and (III) are the basic design chart equations.

APPENDIX III

Specific Compressor Parameters

The pressure coefficient is defined by the following equation where the subscript R , represents values at the tip radius.

$$\psi = \frac{2\eta(\Delta C_v)_R}{U_R} \qquad (5)$$

Since the charts are concerned with constant rotor work axial flow compressors, the following substitution may be made.

$$r\Delta C_v = CONSTANT$$

$$(\Delta C_v)_R = \Delta C_v R_m/R = \Delta C_v(1+\nu)/2$$

also,

$$U_R = UR/R_m = 2U/(1+\nu)$$

substituting these expressions into equation (5) gives,

$$\psi = \eta(1+\nu)^2 \Delta C_v/2U$$

and in terms of the blade load factor and dimensionless rotational speed

$$\psi = \eta \frac{(1+\nu)^2 a}{2b} \qquad (IV)$$

The flow coefficient is defined by the following equation,

$$\phi = V/U_R$$

Substituting the expression for U_R given above

$$\phi = V(1+\nu)/2U$$

and in terms of the dimensionless rotation speed and axial velocity

$$\phi = \frac{(1+\nu)d}{2b} \qquad (V)$$

The specific speed is defined by the following equation:

$$n_s = \frac{N\sqrt{\psi}}{H^{3/4}}$$

It is given by Keller (Axial Flow Fans, 1937) in dimensionless form as follows:

$$n_s = \phi^{1/2}(1-\nu^2)^{1/2}\,\eta^{3/4}\psi^{-3/4} \tag{6}$$

Th a specific speed can be defined in terms of the mean radius velocity triangle by substituting equation (IV) and (V) into equation (6)

$$n_s = \left[\frac{(1-\nu)\,d\,(2b)^{1/2}}{(1+\nu)a^{3/2}}\right]^{1/2} \tag{VI}$$

The throttle number is defined by the following equation

$$N_T = \phi^2/\psi$$

and in terms of the mean radius velocity triangle

$$N_r = \frac{d^2}{2\eta a b} \tag{VII}$$

APPENDIX IV

Generalizing the Chart Parameters

Let quantities without subscripts denote initial, uncorrected values; quantities with subscript s denote the values converted to standard atmospheric conditions; and quantities with the subscript n be values converted to both standard atmospheric conditions and 1 hp power-input.

    a) Conversion to Standard Atmospheric Conditions:

In reversible adiabatic changes of state, the temperature and pressure ratios are uniquely determined by the Mach number. Compressor performance charts are customarily generalized by holding the pressure ratio and, hence, the Mach number constant for varying atmospheric and power conditions. The design charts with which this report is concerned are generalized in the same manner.

To maintain constant Mach number, the rotational speed must vary in accordance with the speed of sound.

$$\text{i.e.} \quad \frac{N}{C_s} = \frac{N_s}{(C_s)_s}$$

The speed of sound, C_s, varies with the square root of the temperature.

$$\frac{N}{(C_s)_s \sqrt{\frac{T}{T_s}}} = \frac{N_s}{(C_s)_s}$$

$$N_s = \frac{N}{\sqrt{\theta}}$$

(7)

The value of p/p_o is held constant for all atmospheric conditions. The design charts of this report, however, are concerned with Δp rather than p/p_o. The general representation of Δp is found as follows:

$$\text{From} \quad \frac{p}{p_o} = \frac{p_s}{p_{o_s}} \quad,$$

$$\frac{p}{p_o} - 1 = \frac{p_s}{p_{o_s}} - 1 \quad, \text{ or}$$

$$\frac{\Delta p}{p_o} = \frac{\Delta p_s}{p_{o_s}}$$

Multiplying both sides by P_{o_2} gives

$$\Delta P_1 = \Delta P \frac{P_{o_1}}{P_o} \text{, or}$$

$$\Delta P_1 = \frac{\Delta P}{\delta} \qquad (8)$$

The axial velocity is proportional to the rotational speed and hence must also vary with $\sqrt{\delta}$. The power input is the product of the axial velocity, the pressure increase and the annulus area. The annulus area is independent of atmospheric conditions and, therefore, the power input must vary with the product of the axial velocity and the pressure increase. That is:

$$P_1 = \frac{P}{\sqrt{\theta}\,\delta} \qquad (9)$$

The tip radius, R , is independent of the atmospheric conditions and hence,

$$R_1 = R$$

b). Conversion to 1 hp power input:

The total pressure increase, ΔP, is not affected by changes in power input and, therefore,

$$\Delta P_n = \frac{\Delta P}{\delta}$$

The rotational speed varies inversely as the square root of the power input.

$$N_n = N_1 \sqrt{P_1}$$

Substituting equation (7) and (9) gives

$$N = \frac{N}{\sqrt{\theta}} \sqrt{\frac{P}{\sqrt{\theta}\,\delta}} \text{ , or}$$

$$N_n = N\theta^{-\frac{3}{4}}P^{\frac{1}{2}}\delta^{\frac{1}{2}}$$

The tip radius varies as the square root of the power input.

$$R_n = \frac{R}{\sqrt{P_1}}$$

Substituting equation (9) gives

$$R_n = \frac{R\,\theta^{\frac{1}{4}}\delta^{\frac{1}{2}}}{P^{\frac{1}{2}}}$$

In summary, the relations for converting from design values to normal values for use with the charts are as follows:
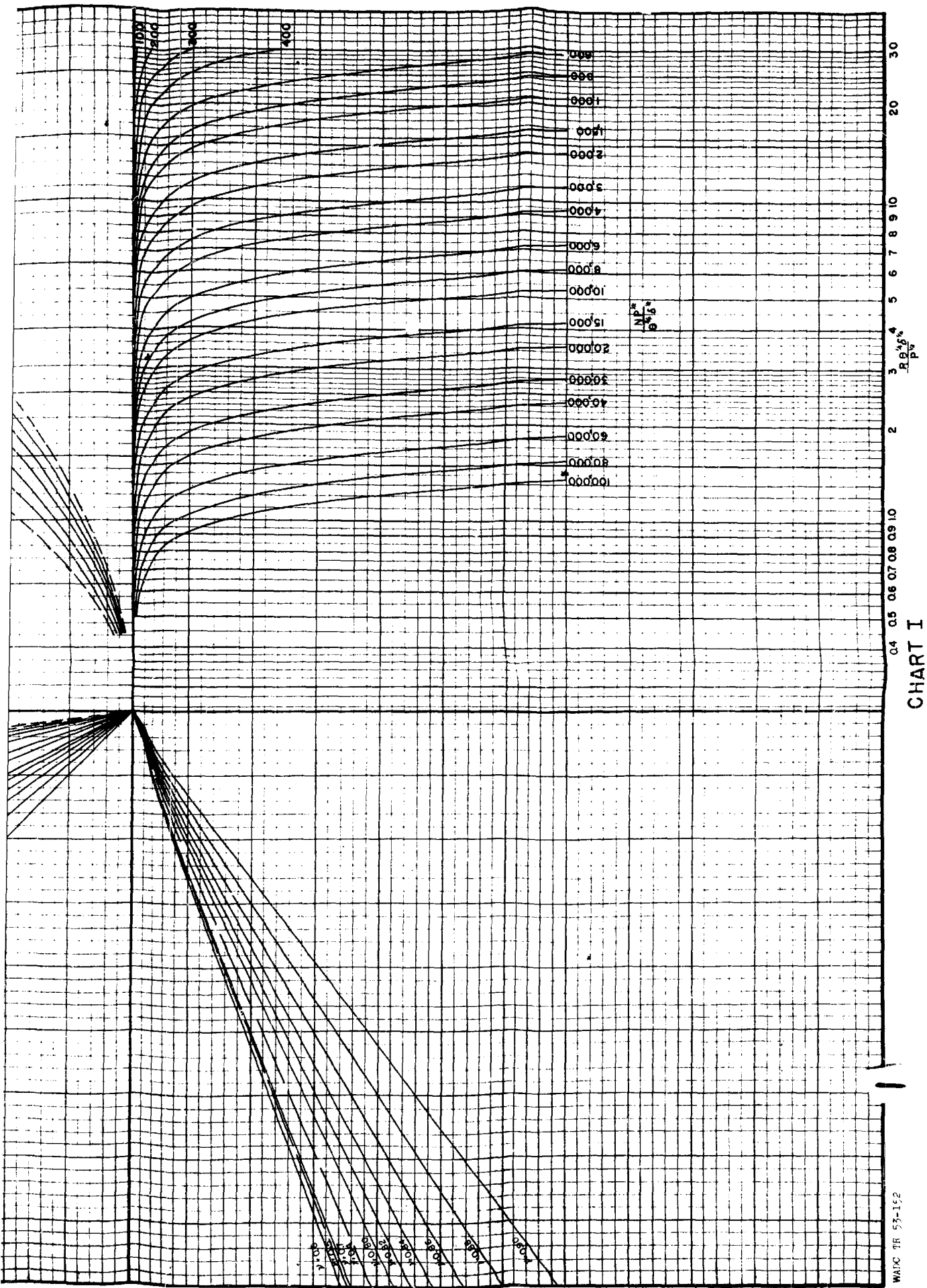
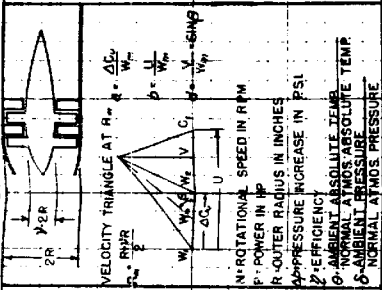$$P_n = 1 \text{ H.P.}$$

$$N_n = \frac{N P^{\frac{1}{2}}}{\theta^{\frac{1}{2}} \delta^{\frac{1}{2}}}$$

$$\Delta p_n = \Delta p / \delta$$

$$R_n = \frac{R \theta^{\frac{1}{2}} \delta^{\frac{1}{2}}}{P^{\frac{1}{2}}}$$

For convenience, the charts are labeled as functions of the actual conditions rather than the normal conditions.

CHART I

$$\frac{N\,P_s^*}{\theta^{*} i_s^{*}}$$
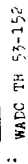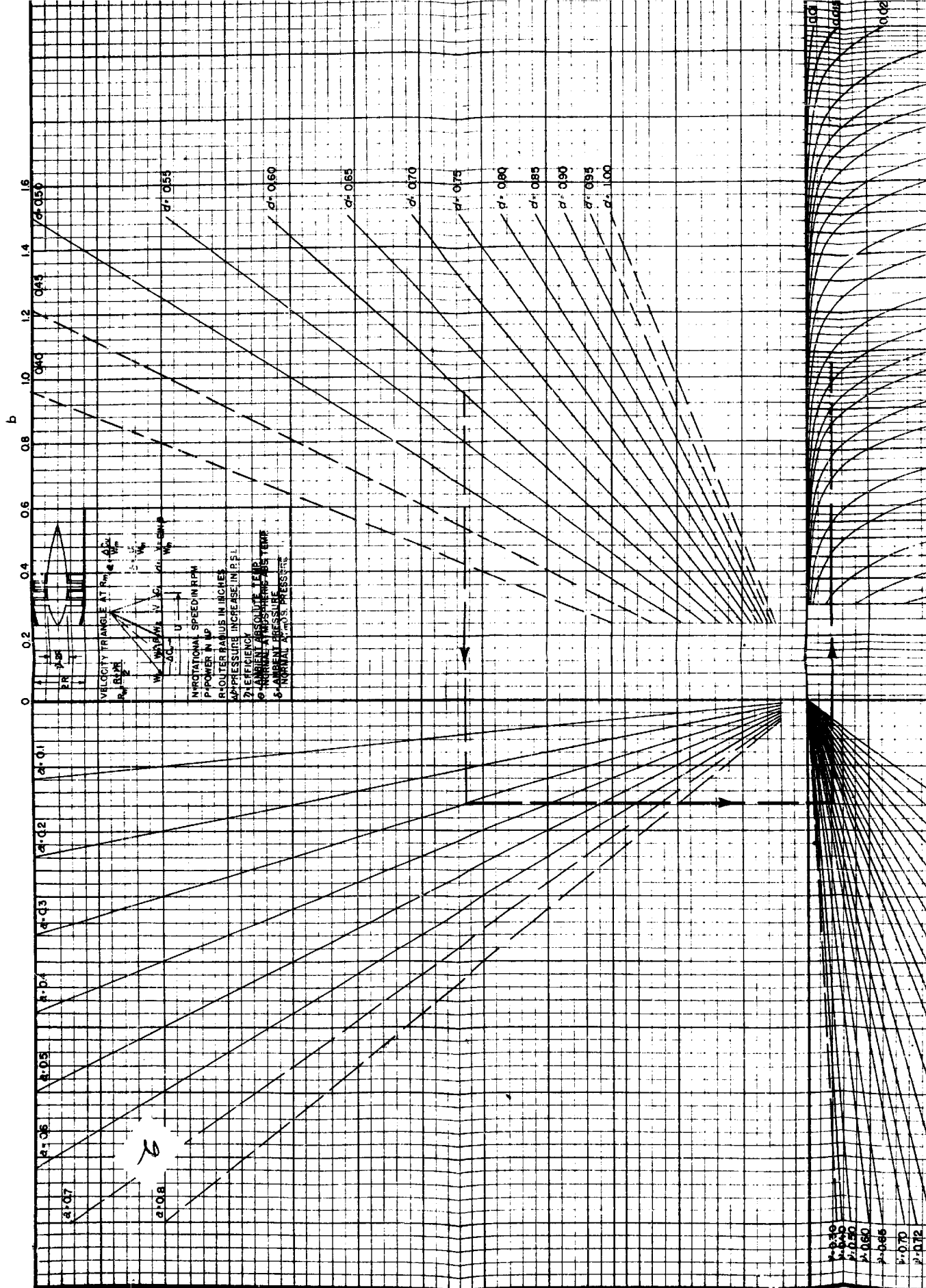
$$\frac{R\,\theta^{*} i_s^{*}}{P_s^{*}}$$

b

VELOCITY TRIANGLE AT R_m

$a = \dfrac{\Delta C_u}{W_m}$

$b = \dfrac{U}{W_m}$

$d = \dfrac{V}{W_p} = \dfrac{i\sqrt{\theta}}{N_p\sqrt{\theta}}$

N = ROTATIONAL SPEED IN RPM
P = POWER IN HP
R = OUTER RADIUS IN INCHES
Δp = PRESSURE INCREASE IN PSI
η = EFFICIENCY
θ = AMBIENT ABSOLUTE TEMP.
θ = NORMAL ATMOS ABSOLUTE TEMP.
δ = AMBIENT ATMOS. PRESSURE
δ = NORMAL ATMOS. PRESSURE

d = 0.4
d = 0.5
d = 0.6
d = 0.7
d = 0.8
d = 0.9
d = 1.0

a = 0.14   a = 0.16   a = 0.2   a = 0.25   a = 0.3   a = 0.4   a = 0.5, 0.6, 0.7, 0.8
a = 0.12
a = 0.18
a = 0.10

CHART II

VELOCITY TRIANGLE AT P_m

N=ROTATIONAL SPEED IN RPM
P=POWER IN HP
R=OUTER RADIUS IN INCHES
ΔP=PRESSURE INCREASE IN P.S.I.
η=EFFICIENCY
θ=AMBIENT ABSOLUTE TEMP NORMAL ATMOSPHERE TEMP
δ=AMBIENT PRESSURE NORMAL ATMOS. PRESSURE

CHART III

VELOCITY TRIANGLE AT ...

N = ROTATIONAL SPEED IN RPM
P = POWER IN HP
R = OUTER RADIUS IN INCHES
= PRESSURE INCREASE PSI
= EFFICIENCY
= AMBIENT ABSOLUTE TEMP / NORMAL ATMOS ABSOLUTE TEMP
= AMBIENT PRESSURE / NORMAL ATMOS PRESSURE

POSITIVE PRE-ROTATION

NEGATIVE PRE-ROTATION

(CURVED LINES ARE ZERO PREROTATION LINES FOR THE INDICATED VALUES OF a)

$d=0.09$ $d=0.1$ $d=0.08$ $d=0.07$ $d=0.6$ $d=0.5$ $d=0.4$

$a=0.10$ $a=0.12$ $a=0.14$ $a=0.16$ $a=0.18$ $a=0.2$ $a=0.22$ $a=0.25$ $a=0.3$ $a=0.35$ $a=0.4$ $a=0.45$ $a=0.5$ $a=0.55$ $a=0.6$ $a=0.7$ $a=0.8$

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